Title: Applications of Sine and Cosine Graphs

Standard(s):

MA3A3. Students will investigate and use the graphs of the six trigonometric functions.

a. Understand and apply the six basic trigonometric functions as functions of real numbers.

b. Determine the characteristics of the graphs of the six basic trigonometric functions.

Essential Question(s): How are Sine and Cosine graphs applicable in real life?

Materials: Poster board, computer, graphing calculator, markers

Mini-Lesson: *At this point in the unit, students should be familiar with the graphs of the functions of sine and cosine. In addition, they should be comfortable with the different transformations done to the parent functions* $f(x) = sin(x)$ *and* $f(x) = cos(x)$*, both in writing the equations from given information, and sketching a picture of what the graphs look like.*

To begin this lesson, the teacher will introduce a guest speaker. This guest speaker can be anyone whose career uses trigonometry on a daily basis. Possible guests could include an engineer, a video game creator, an architect, etc. The guest will give a short (10-15) minute presentation modeling how they use sine and cosine in their everyday life. Following the presentation there will be a 5 min Q & A. Once the guest is done, the teacher will introduce the day(s) objective.

Work Session: Students will be placed in groups no larger than four students. These groups will be organized by not only ability but also interest. During the previous class, the teacher will give the students an interest-alyzer to better assist in grouping. The students will need to pick an area or object of interest from, music, ferric wheel, weather, bicycle, trampoline. Students may also find their own and get it approved by the teacher. Students will then gather information on how sine and cosine are used in the specific area or with the particular object. Students will have time to conduct research on the laptops to get a better understanding of the area or object as well as the mathematics behind it. Once the students have gathered enough information, they will be given a particular set of questions that pertain to their topic. The students will answer these questions as a group and then prepare to present their findings. The presentations should include a visual aide such as a poster or prezi, which showcases the background information they learned as well as the mathematics they completed.

Closing: To summarize this lesson, students will complete a gallery walk of sorts. Groups for the gallery walk will be formed so that at least one member from each group is present in a group. As the groups move around the room looking at the visual aids, there will be at least one person who worked on the topic so that they can provide more information or clarification.

Differentiation: Students will be grouped based on ability in addition to interest. Students in the group will be with peers on the similar level of ability. The higher ability students will be expected to have a more mathematically rich product while those who are lower ability will be expected to complete the questions provided.

Assessment: The gallery walk will serve as the assessment for this lesson, in addition to the monitoring the teacher will do throughout the work period(s). Having one member from each topic in a group during the walk will hold each member accountable for the final product.

Background information on students: This class is an Accelerated Math III course. There is a mixture of identified gifted students, high ability (non identified gifted students) and on level students.

Discussion: The goal of this lesson is to allow students to see how the sine and cosine functions are helpful in modeling real world phenomena as well as help make connections between the four transformations, amplitude, period, phase shift, and vertical shift and how that looks in real situations. To begin, having a guest speaker acts a hook to peak the interest of the students. Sometimes as a teacher, you can tell them this and that math is applicable but until students see just how applicable it is (not from their math teacher) they struggle to see the value in the concepts. This type of "hook" follows closely with Dr. Renzulli' School Wide Enrichment Model, as a type I experience. In addition, following the Incubation model created by Dr. Torrance, this would act as a Stage 1 activity to heighten anticipation. Once the groups are formed, the individuals will have the responsibility and freedom to choose a topic or object of their choice. The beginning stage of this mini-project will support the students' development of research skills and ultimately allow them to develop a product to be presented. This mini-project could also serve as an opportunity for a group to do the preliminary stages of a science fair project or a personal project.

References:

Georgia Department of Education (2011). *Mathematics Georgia performance standards*. Retrieved from https://www.georgiastandards.org/Common-Core/Pages/Math-9-12.aspx

Hebert, T., Cramond, B., Neimuster, K., Millar, G., & Silvan, A. (2002). E. paul torrance: His life, accomplishments, and legacy. *THE NATIONAL RESEARCH CENTER ON THE GIFTED AND TALENTED*, Retrieved from http://www.gifted.uconn.edu/nrcgt/reports/rm02152/rm02152.pdf

Neag. (n.d.). *Sem movies*. Retrieved from http://www.gifted.uconn.edu/sem/semmovie.html

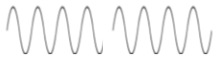
Sample problems ideas: Retrieved from GA BOE Accelerated Math III Unit 5

Applications of Sine and Cosine Graphs Learning Task:

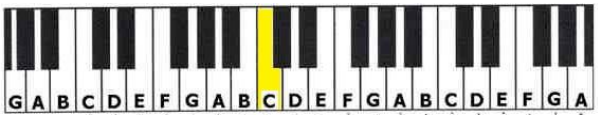
1. Musical Tones

There is a scientific difference between noise and pure musical tones.

A random jumble of sound waves is heard as noise.	
Regular, evenly spaced sound waves are heard as tones.	

The closer together the waves are the higher the tone that is heard.	
The greater the amplitude the louder the tone.	

Trigonometric equations can be used to describe the initial behavior of the vibrations that give us specific tones, or notes.



a. Write a *sine* equation that models the initial behavior of the vibrations of the note G above middle C given that it has amplitude 0.015 and a frequency of 392 hertz.

b. Write a *sine* equation that models the initial behavior of the vibrations of the note D above middle C given that it has amplitude 0.25 and a frequency of 294 hertz.

c. Based on your equations, which note is higher? Which note is louder? How do you know?

d. Middle C has a frequency of 262 hertz. The C found one octave above middle C has a frequency of 254 hertz. The C found one octave below middle C has a frequency of 131 hertz.

   i. Write a *sine* equation that models middle C if its amplitude is 0.4.

   ii. Write a *sine* equation that models the C above middle C if its amplitude is one-half that of middle C.

   iii. Write a *sine* equation that models the C below middle C if its amplitude is twice that of middle C.

The Ferris Wheel

There are many rides at the amusement park whose movement can be described using trigonometric functions. The Ferris Wheel is a good example of periodic movement.

Sydney wants to ride a Ferris wheel that has a radius of 60 feet and is suspended 10 feet above the ground. The wheel rotates at a rate of 2 revolutions every 6 minutes. (Don't worry about the distance the seat is hanging from the bar.) Let the center of the wheel represents the origin of the axes.

a. Write a function that describes a Sydney's height above the ground as a function of the number of seconds since she was ¼ of the way around the circle (at the 3 o'clock position).

b. How high is Sydney after 1.25 minutes?

Weather Models

A city averages 14 hours of daylight in June, 10 in December, and 12 in both March and September. Assume that the number of hours of daylight varies periodically from January to December. Write a cosine function, in terms of t, that models the hours of daylight. Let **t** = 0 correspond to the month of January.

More Applications

The inside of a bicycle wheel whose diameter is 25 inches is 3 inches off the ground. An ant is sitting on the inside of the wheel. Steve starts riding the bicycle at a steady rate. In 1.2 seconds the ant reaches its highest point on the wheel. The wheel makes a revolution every 1.6 seconds. Determine 5 points and find the equation that describes the motion of the ant.

a) What will be the height of the ant 25 seconds into the ride?

b) Within the first 10 seconds, how many times will the ant be at its starting height?

John is bouncing up and down on a trampoline which is 4 feet off the ground. The highest he gets off the trampoline is 11 feet which he reaches in 2 seconds. He completes a bounce every 3 seconds. Determine 5 points and find the equation that describes this motion.

a) In 45 seconds, what will be his height? Is he going up or down at this time?

b) Within the first 45 seconds, how many times does he reach his peak?

Sample's from my class who did the lesson.